Name: $\qquad$ ID $\qquad$

Instructions:

- This exam contains four questions with multiple parts.
- Time allowed: 180 minutes
- Closed Book, Closed Notes.
- There are 10 pages in this exam booklet.
- Use of Calculators and / or computing devices / smartphones etc is strictly prohibited.
- Answer the problems on the exam sheets only. No additional attachments would be accepted.
- When the "time is over" is called, it is students responsibility to submit his exam to the invigilator. Submitting completed exam 3 minutes after the "time is over" will incur a penalty of 5 points.

Few gentle reminders:

- If you get stuck on some problem for a long time, move on to the next one.
- The ordering of the problems is somewhat related to their relative difficulty. However, the order might be different for you!
- You should be better off by first reading all questions and answering them in the order of what you think is the easiest to the hardest problem.
- Keep the points distribution in mind when deciding how much time to spend on each problem.

Do Not write below this line:

| a | b | a | b | a | b | a | b | Total: |
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| C |  | C | d | C |  | C |  |  |

CLO Assessment

| CLO1 - Q1(10) | CLO2 - Q2 (10) | CLO3 - Q4 (10) | CLO4 - Q3a (4) |
| :--- | :--- | :--- | :--- |

Q1 (a) [2]. Give the Big-Oh notation for the following functions

| $f(N)=N^{2} \log \mathrm{~N}^{2}+2 \mathrm{~N} \log 2 \mathrm{~N}$ |  |
| :--- | :--- |
| $\mathrm{f}(\mathrm{N})=\left(\mathrm{N} \cdot\left(100 \mathrm{~N}+5+\mathrm{N}^{3}\right)\right)^{2}$ |  |
| $\mathrm{f}(\mathrm{N})=\mathrm{N}^{1 / 2}+\log \log \mathrm{N}$ |  |
| $\mathrm{f}(\mathrm{N})=1000 \log \log \mathrm{~N}+\log \mathrm{N}$ |  |

Q1 (b) [4]. Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable $\boldsymbol{n}$.

```
public int silly(int n, int m) {
    if (n < 1)
        return m;
    else if (n < 10)
        return silly(n/2, m);
    else
        return silly(n - 2, m);
}
public void warm(int n) {
    for (int i = 0; i < 2 * n; ++i) {
        j = 0;
        while (j < n) {
        System.out.println("j = " + j);
        j = j + 5;
        }
    }
}
```

Q1 (c) [4]. Describe the most time-efficient way to implement the operations listed below. Give the runtime for your procedure in Big-Oh notation.

| Push a value onto a stack implemented as an array. Assume the array is of size 2 N. |  |
| :--- | :--- |
| Searching for all edges starting at vertex v in a edge-list for a Graph |  |
|  |  |
| Given a binary min-heap, remove the minimum element from this heap |  |

Q2 (a) [2].
What is the minimum and maximum number of nodes in a binary tree with height $\mathbf{6}$ ?

Min:

Max:

Lets say the height of a binary tree is h . What is the maximum number of leaf nodes in this tree?

Q2 (b) [3] Illustrate a step-by-step result of inserting the following values in a binary max-heap. 8, 3, 4, 5, 1, 2, 10.

Q2 (c) [3] Remove the key 19 from the AVL tree given below. Balance the tree after the removal operation. Show all intermediate steps highlighting rotations.


Q2 (d) [2]. Observe the binary trees given below:

(d)

Write only the letters of appropriate trees. Which of the above are: (Note: It is possible that none of the trees above have the given property, it is also possible that some trees have more than one of the following properties.)

AVL Tree? $\qquad$

Complete Tree? $\qquad$

Binary heap? $\qquad$

Q3 (a) [4]. Draw the contents of the two hash tables below after inserting the values shown. Show your work for partial credit. If an insertion fails, please indicate which values fail and attempt to insert any remaining values. Use linear probing for collision resolution.
$H(k)=k$ mod array_size

Insert: 12, 5, 19, 2, 23


Insert: 9, 18, 19, 27, 8, 17


Q3 (b) [4]: Draw the adjacency list and adjacency matrix for the following directed graph.


Adjacency Matrix

Adjacency List

Q3(c) [2]: Provide a Depth First Search Traversal and Breadth First Search Traversal for this digraph starting at position 4.

Depth First Search:

Breadth First Search:

Q4 (a) [4]: Apply Natural Merge-sort to the following data. Show all steps as a single processor computer would process this data.

| mid |
| :--- |
| low |
| law |
| pie |
| big |
| paw |
| ate |
| bad |
| sad |

Q4 (b) [4]. Give Big-Oh notation for the following problems. For partial credit explain your algorithm.
Given an array of integers of size $N$, if we use a min-heap to sort integer data in ascending order, what would be the run-time?

Given an array of integer values of size $2^{*} \mathrm{~N}$ where all the data is already sorted in descending order. Give most efficient way of reversing the contents of this array so that the items in array would be sorted in ascending order.

What will be the run time of sorting an integer array with size $N$ if we use an AVL tree for it?

Find the in-degree of a vertex if an adjacency list of a directed graph is given? (use E or V instead of N in your answer)

Q4(c) [2]: Suppose you are managing director of a financial organization called Acme Inc. that holds records of your clients' accounts and bank transactions. At the end of a fiscal month, your company produces a report listing all the clients ordered by
(1) the highest balance per account and
(2) number of transactions per month.

Since you have the technical ability of writing efficient algorithms, you decided to use an array of type BAccount to store the required information, sort the array and produce your report.

Give an efficient algorithm that will allow you to sort this array in the required order (stable). What would be the run time of your algorithm? Following is the structure of the class BAccount.

```
public class BAccount{
    public String CustomerID;
    public int BankAccount;
    public double balance;
    public int transactions
}
```

