$\qquad$ SOLUTION $\qquad$ ID $\qquad$ Weight: 15\% Time: 45 minutes

Solve all Questions
Q1 [2 points] Give the Big-Oh notation for the following functions

| $f(N)=N^{2}+\log \mathrm{N}^{2}+2 \mathrm{~N} \log \mathrm{~N}$ | $\mathbf{O}\left(\mathbf{N}^{2}\right)$ |
| :--- | :--- |
| $\mathrm{f}(\mathrm{N})=(\mathrm{N} \cdot(100 \mathrm{~N}+5000000))^{2}$ | $\mathbf{O}\left(\mathbf{N}^{4}\right)$ |
| $\mathrm{f}(\mathrm{N})=\mathrm{N}^{1 / 2}+\log (\log \mathrm{N})$ | $\mathbf{O}(\mathbf{N})$ |
| $\mathrm{f}(\mathrm{N})=1000^{100}+\log \mathrm{N}$ | $\mathbf{O}(\log \mathrm{N})$ |

Q2. [2 points] Assume you have a non-empty Stack $S$ and a Queue $Q$ of type integers. Provide a sequence of calls to modify the contents of $S$ and $Q$ from scenario_1 to scenario_2.

scenario_1

S


Q

| 4 | 7 | 8 |  |  |
| :--- | :--- | :--- | :--- | :--- |

scenario_2
Q.enqueue (S.pop () )
Q.enqueue (S.pop())
Q.enqueue (S.pop())
S.push (Q.dequeue ())
S.push (Q.dequeue())

Q3 [4]. Given the following code, estimate the number of operations and describe the worst case running time in Big-Oh notation in terms of the variable n. Show trace where appropriate.

| ```public int Mul(int [] A, int i, int t) { if (i >= A.length-1) return t; else return Mul(A, ++i, t * A[i]); } Mul() is recursively called i times. I would run up to n, which is the length of this array. The run time will be O(n)``` | Mul(new int[] $\{1,2,3,4\}, 0,1$ ) |
| :---: | :---: |
| ```public void diag(int n) { int count = 0; for (int i = 0; i < n; i++) for (int j = 0; j < i; j++) if(i==j) count++; } \\ The if statement will execute n/2 * ( \(\mathrm{n}+1\) ) times \[ 1+2+3+4+5+\ldots=\frac{n(n+1)}{2} \] \\ which is \(O\left(n^{2}\right)\)``` |  |

Q4. [3 points] Write an iterative method public static double addDiagonal(double [][] D) that adds and returns all the left-diagonal values of a $\mathrm{n} \times \mathrm{n} 2$-dimensional array.

Show a trace to determine the runtime function $T(n)$ estimating the number of operations.


```
public static double addDiagonal(double [][] D)
{
    double total = 0;
    for(int i=0;i<D.length;i++)
            total+=A[i][i];
    return total;
}
```

We must note that the size of the matrix is $\mathrm{n} x \mathrm{n}$, which means it is a square matrix. We estimate the number of operation to be:
$T(n)=5 n+3$
which is $\mathbf{O}(\mathrm{n})$
[+1 Bonus] Write the above method as a recursive method. Estimate the runtime of this method.

```
public static double addDiagonal(double [][] D, int i)
{
    if(i>=D.length)
        return 0;
    else
        return D[i][i] + addDiagonal(D, ++i);
}
```

We can estimate that the run time $T(n)=5 c n$ which is $O(n)$
adsDiagonal ( $D, 0$ )

* $[0][0]+$ ald $\operatorname{Diagmal}(D, 1)$


Q5. [4 points] Modify the method insert that you defined in the double linked list class DList. The modified method has the following definition.

## public static void insert2(Node $N$, int pos)

This method inserts a node $N$ at position pos in the DList. You may start counting at the head. Make sure your method checks if value of pos is valid (i.e. pos must be a non-negative integer with value less than size).

```
public static void insert2(Node N, int pos){
    //we assume the first position is zero
    if(pos<0 || pos > size+1)
        return;
    if(pos==0)//add to the beginning
    {
        N.next = Head;
        N.prev = null;
        Head.prev = N;
        Head = N;
    }
    else if(pos == size) // we add at the end
    {
        N.next = null;
        N.prev = Tail;
        Tail.next =N;
        Tail = N;
    }
    else
    {
        //we add in the middle
        Node temp = Head;
        for(int i=1;i<pos;i++)
            temp = temp.next;
        N.next = temp.next;
        N.prev = temp.prev;
        temp.next = N;
        N.next.prev = N;
    }
    size++;
    return;
}
```

