There are five questions in this exam. Answer all questions. Time is of the essence, use it wisely!
\{CLO 1\}Q1 [3]. Consider the following multiple choices, circle the correct answer. Each choice is worth 0.5 points.
A. Consider the below single-left rotation pseudo code for an AVL tree where an AVL node contains value pointers to left, right child nodes.

```
AVLNode single_left_rotation(AVLNode z){
AVLNode x,y,a,b,c,d;
y=z.left;
x=y.left;
a=x.left;
b=x.right;
c=y.right;
d=z.right;
return y; }
```

What is missing?
a) y.right $=z$;
b) z left $=\mathrm{c}$
c) y.right $=z ;$ z.left $=c$;
d) None of the above
B. For the code snippet given in part A, what is the height at node $y$ ?
a) Math.max(y.height - x.height) +1
b) $y$.height $=x$. height +1 ;
c) $y$.height $=$ z.height +1 ;
d) None of the above
C. In a binary max-heap implementation using an array, what is the position of a parent node for an arbitrary node $x$; assume the index starts at 1 .
a) (i/2) position
b) $(i+1) /$ position
c) Math.floor(i/2) position
d) Math.ceil(i/2) position
D. Given an array of element $5,7,9,1,3,10,8,4$. Tick all the correct sequences of elements after inserting all the elements in a min-heap.
a) $1,3,4,7,8,9,10$
b) $1,4,3,8,9,5,7,10$
c) $1,3,4,5,8,7,9,10$
d) None of the mentioned
E. What are the worst case and average case complexities of a binary search tree?
a) $O(n), O(n)$
b) $O(\log n), O(\log n)$
c) $O(\log n), O(n)$
d) $O(n), O(\operatorname{logn})$
F. What is the worst-case time complexity to delete an element from a hash-table implemented with separate-chaining?
a) $O(n)$
b) $O(\log n)$
c) $O($ nlogn $)$
d) $\mathrm{O}(1)$
$\{$ CLO 2\}Q2. [3]. Sort the array of integers $=\{\mathbf{7 , 5 , 6}, \mathbf{1 1}, \mathbf{1 0}, \mathbf{9}, \mathbf{1 6}, \mathbf{1 5}, \mathbf{1 3}, \mathbf{1 2}, \mathbf{8}, \mathbf{3}\}$ using quick sort. Show all sequences of Pivots during partitioning.
\{CLO 3\}Q3. [3]. Assume you are using a programming language that allows different data types to be stored in a single array. Given an array of $N$ elements of three different types: Cold, Warm, and Hot; design and describe clearly an in-place algorithm to put all the cold elements, on the left, followed by all the warm elements, followed by all the hot elements on the right. Your algorithm can use only a small constant amount of extra space. What is the run time? Show how your algorithm would operate on this array:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | W | H | C | W | W | C | H | C | W | C |

$\{\mathrm{CLO} 4\}$ Q4. [3] Consider a hash table of size 9 that stores entries with integer keys. This hash table uses a double hash function
$h(k)=k \bmod 11$
$f(h(k))=k \bmod 7$
Insert, in the given order, entries with keys $\mathbf{8}, \mathbf{1 1}, \mathbf{1 8}, \mathbf{2 2}, \mathbf{2 8}, \mathbf{1 3}, 25$ into the hash table using linear probing to resolve collisions. Show all the work and fill the array $\mathbf{A}$ accordingly.
A


| $k$ | $h(k)$ | $f(h(k))$ | Collisions |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

What is the number of collisions?
\{CLO 1\}Q5. [3] Remove the following keys from the Binary search tree given below. Illustrate the tree after each removal. Identify which case applies?

13, 32, 19.


