## CS210 Data Structures <br> (221) Final Exam

Name: $\qquad$ ID $\qquad$

Check your section:

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Instructions:

- This exam contains four questions with multiple parts, on 6 sheets of papers (2 sided).
- Time allowed: 180 minutes
- Closed Book, Closed Notes.
- Use of Calculators is ALLOWED. Use of other computing devices / smartphones etc is strictly prohibited.
- Answer the problems on the exam sheets only. No additional attachments would be accepted.
- When the "time is over" is called, it is the students' responsibility to submit his exam to the invigilator. Submitting completed exam 3 minutes after the "time is over" will incur a penalty of $\mathbf{5}$ points.

Few gentle reminders:

- If you get stuck on some problem for a long time, move on to the next one.
- You should be better off by first reading all questions and answering them in the order of what you think is the easiest to the hardest problem.
- Keep the points distribution in mind when deciding how much time to spend on each problem.

| Question No. | Part a | Part b | Part c | Part d | Part e | Student's Score |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Question 1 <br> (CLO 1) | $/ 4$ | 13 | $/ 3$ | $/ 3$ | $/ 3$ | $/ 16$ |  |
| Question 2 <br> (CLO 2) | 12 | 12 | $/ 4$ |  |  |  | $/ 8$ |
| Question 3 <br> (CLO 3) | 14 | 14 |  |  |  | $/ 8$ |  |
| Question 4 <br> (CLO 4) | $/ 4$ | 14 |  |  |  | $/ 8$ |  |
| Total |  |  |  |  |  |  | $/ 40$ |

Question 1. $[4+3+3+3+3=16$ points] [CLO 1]


Part a. [2+1+1 points] Show the adjacency list for the directed graph illustrated above. Do a Depth First Search (DFS) and Breadth First Search (BFS) starting at 0 . Choose the smaller value on vertices to select the next destination/hop.

Part b. [3 points] Observe the following AVL Tree.


Remove 47 from the above tree. Re-Draw the tree and show appropriate rotations.

From the resulting tree you drew, remove 6. Re-Draw the tree and show appropriate rotations

From the resulting tree you drew above, insert 28. Re-Draw the tree and show appropriate rotations

Part c. [3 points] Draw the max-heap given in the following array Show the heap operations for removing two values from this max-heap. Re-draw the heap after each removal.

| 0 | ${ }^{1}$ | ${ }^{2}$ | ${ }^{3}$ | ${ }^{4}$ | ${ }^{6}$ | ${ }^{6}$ | ${ }^{2}$ | ${ }^{2}$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{2 8}$ | $\mathbf{2 1}$ | $\mathbf{2 3}$ | $\mathbf{1 9}$ | $\mathbf{1 8}$ | $\mathbf{1 6}$ | $\mathbf{1 2}$ | $\mathbf{8}$ | $\mathbf{1 0}$ | $\mathbf{6}$ | $\mathbf{1}$ |

Part d. [3 points] An AVL Tree is essentially a balanced BST. Write a method Node Ancestors (int $\mathbf{x}$ ) that takes a parameter int $\mathbf{x}$; this method finds the node N with value x . It returns a String containing all the ancestors of node N separated by spaces. Assume the tree stores integer Key in each node.

String Ancestors (int x) \{

Example:


A call Ancestors(21) for this BST would return " $11,41,3,77$ "

Part e. [3 points] Write a method called OddsToStack that takes as parameter, a singly-linked list L, which contains an integer value in each node. It returns a stack that contains all the elements of the list except even numbers. Note the order of items is reversed!

public Stack OddsToStack (SinglyLinkedList L) \{

Question 2. [2 $+2+4=8$ points] [CLO 2]
Part a. [2 points] The following methods search for an integer key in a 3D Array and returns true if found.
Give their run-time.


Both programs are identical, which is better and why?

Part b. [2 points] Consider the following recursive method mul that takes two parameters int start and int end. The method multiplies all integer values between start and end returning the result as an integer value. Describe the worst-case running time of the function mul. Show/draw the recursion trace as necessary for this call.

```
public static int mul(int start, int end) {
    if (start > end)
        return 1;
    else
        return end * mul(start, end -1)
}
```

Part c. [4 points] Identify the correct choice from the following MCQs:

|  | Choice? |
| :---: | :---: |
| The runtime cost of a double left/right rotation in an AVL tree is <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | a |
| The runtime cost of deleting a key from the AVL tree is <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $O(n \log n)$ | b |
| Space required for storing an Un-directed Graph in an Adjacency Matrix is <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\mathrm{N})$ <br> c. $\mathrm{O}(\mathrm{V} * \mathrm{E})$ <br> d. $\mathrm{O}(\mathrm{V} * \mathrm{~V})$ | d |
| Removing an edge from a given edge-list for a directed graph take this runtime: <br> a. $\mathrm{O}(\mathrm{E})$ <br> b. $\mathrm{O}(\mathrm{E} * \mathrm{E})$ <br> c. $\mathrm{O}(\mathrm{V} * \mathrm{E})$ <br> d. $\mathrm{O}(\mathrm{V} * \mathrm{~V})$ | a |
| The runtime cost of traversing and printing all node values in an AVL tree is <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | c |
| Worst case scenario for searching the smallest value in a AVL Tree <br> a. $\mathrm{O}(\mathrm{n} 2)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | b |
| Searching a value in a min-heap <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | c |
| Removing a node at the Tail of a doubly linked list <br> a. $\mathrm{O}(1)$ <br> b. $\mathrm{O}(\log \mathrm{n})$ <br> c. $\mathrm{O}(\mathrm{n})$ <br> d. $O(\mathrm{n} \log \mathrm{n})$ | a |

Question 3. [6+2 = $\mathbf{6}$ points] [CLO 3]
Part a. [6 points] Suppose you have to store the following values in a hash table, implemented using linear probing. The hash function used was the identity function, $\mathbf{h}(\mathbf{x})=\mathbf{x} \bmod 13$. Assume that the hash table has a size is of 10 ; insert the following in this hash-table.

9, 27, 39, 16, 22, 35, 18, 8, 20, 28.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |

Identify, how many collisions $\qquad$ and displacements $\qquad$ occurred.

The table size is 10 . If we re-size it to 13 , would it make any difference?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
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Identify, how many collisions $\qquad$ and displacements $\qquad$ occurred.

Assuming the same data is inserted in a Hash-table size 13, using separate chaining as a collision resolution method, Draw the Hash table.

| 0 |
| :--- |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |

Identify, how many collisions $\qquad$ and displacements $\qquad$ occurred.

Part b. [2 points] Identify the correct answers from these MCQs
What is the Big O run-time for searching a value in a hash-table with linear probing
a. $O(1)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}(\mathrm{n})$
d. $O(n \log n)$

One of these is not a correct answer for reducing collisions in a hash table
b
a. Use Linear probing method
b. Use random values for hashing
c. Use Separate Chaining method
d. Use uniform hashing method

This data structure is useful for separate chaining
c
a. Stacks
b. Queue
c. Linked Lists
d. Heaps

A poor hash function would result in a hash table with:
a. Clustering
b. Uniform hashing
c. No collisions
d. No displacements

Question 4. [4 $+4=8$ points] [CLO 3]
Part a. [4 points] Show the trace how the Mergesort algorithm sorts the given array, Identify the mid points for each pass/iteration.

| $\mathbf{K}$ | $\mathbf{R}$ | $\mathbf{A}$ | $\mathbf{T}$ | $\mathbf{E}$ | $\mathbf{L}$ | $\mathbf{E}$ | $\mathbf{P}$ | $\mathbf{U}$ | $\mathbf{I}$ | $\mathbf{M}$ | $\mathbf{Q}$ | $\mathbf{C}$ | $\mathbf{X}$ | $\mathbf{O}$ | $\mathbf{S}$ |
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Part b. [4 points] Give the correct answer for the following MCQs

Given the following sequence: $\{2,3,5,6,9,11,15\}$. Which sorting algorithm will run in least time ( n comparisons)?
a. Insertion Sort
b. Selection Sort
c. Heap Sort
d. Merge Sort

Number of swaps/exchanges occurred sorting the data $\{0,1,2,0,1,2,0,1,2\}$ using Selection sort.
a. $\mathrm{O}(1)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}(\mathrm{n})$
d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$

Number of swaps/exchanges occurred sorting the data $\{0,0,0,1,1,1,2,2,2\}$ using Insertion sort.
a. $\mathrm{O}(1)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}(\mathrm{n})$
d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$

Worst-case run-time for quick-sort
a. $\mathrm{O}\left(\mathrm{n}^{2}\right)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}\left(\mathrm{n}^{3}\right)$
d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$

Worst-case run-time for bottom-up merge-sort
D
a. $\mathrm{O}\left(\mathrm{n}^{2}\right)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}\left(\mathrm{n}^{3}\right)$
d. $\mathrm{O}(\mathrm{n} \log \mathrm{n})$

Which if the following sorting algorithms requires extra space (i.e. not in-place)
a. Insertion Sort
b. Selection Sort
c. Quick Sort
d. Merge Sort

Running mergesort on an array that is already sorted takes this time:
D
a. $\mathrm{O}(1)$
b. $\mathrm{O}(\log \mathrm{n})$
c. $\mathrm{O}(\mathrm{n})$
d. $O(n \log n)$

Which of the following algorithm design techniques is used in the quick sort algorithm
a. Backtracking
b. Divide and Conquer
c. Uniform hashing
d. None of the above
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